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A Study of Equilibrium Joining and Socially Optimal Strategic Behavior in Discrete-Time $GI^{[x]}/Geo/1$ Queue with Multiple Working Vacations

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Abstract: We analyze equilibrium balking strategies and the social benefits in a discrete-time batch arrival renewal input queue with multiple working vacations. The arriving batch customers decide whether to join or balk the queue on the basis of a natural reward-cost structure, which integrates their wish for service as well as their reluctance to wait. We study customers' behavior and estimate the net benefit of the batch customers that prefer to take part in the system. The fully observable and unobservable cases with respect to various levels of information availability from the system are examined. We analyze the fully observable case by applying embedded Markov chain and the fully unobservable case by using the roots of the characteristic equations of the probability generating function of system length distribution at pre-arrival epochs. The significance of the information levels along with various parameters on the equilibrium behavior and social benefits is illustrated by numerical results.

Keywords: Batch arrival, discrete-time, equilibrium strategies, renewal input,working vacation, social welfare.

1.Introduction

An equilibrium and social optimal behavior of customers in queueing systems are widely used to analyze from an economic perspective. In the case of balking, the decision to join or not to join the queue is left to the customers which are more reasonable than the classical queueing theoretic viewpoint where customers are compelled to pursue the decisions taken by the system. Information about the system is made available to the customers at their arrival, which helps the customers in decision making, based on the customer's perception about the system congestion. Unlike the classical queueing theory, where customers are ignorant about the system, with balking features, the customers are having some knowledge about the system at the time of arrival. Particularly, some reward-cost structure is imposed on the system that expresses the customers' goal for service and their unwillingness for waiting. With the increase of joining rate of the customer, its delay also increases, which sets a negative effect on the system.

The information pertaining to system-length and server's status is made available to customers at their arrival epoch. We discuss two different cases: (i) fully observable: the information about the system-length and server's status is provided to arriving customers; (ii) fully unobservable: the arriving customer remains uninformed about the system-length and the server's status. The equilibrium balking strategies for M/M/1 queue are analyzed with various types of consideration: Burnetas and Economou [4] (with setup time), Zhang et al. [16] (for multiple working vacations (MWV)). Equilibrium balking strategies in discrete-time queueing systems have been reported in Ma et al. [15] (with multiple vacations), Bixuan et al. [3] (with MWV), Wang et al. [13] (for single working vacation). A single working vacation for renewal input queueing system has been investigated by AD Banik [2]. The renewal input discrete-time*GI/Geo/*1

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queue with multiple working vacation is analyzed by Li et al. [11]. Laxmi et al. [12] considered *GI/Geo/1/N* queue with balking and multiple working vacations. A comprehensive analysis on customer impatience for M/M/1, M/G/1, and M/M/c queue has been analyzed by Altman and Yechiali [1]. State-dependent balking, reneging and server breakdown for M/M/R queue has been analyzed by Wang and Chang [14]. Analysis of Nash equilibrium behavior of customers for multiple and single vacation policy in discrete-time queue is reported in [10]. Similar analysis for the observable *Geo/Geo/1* queue with delayed multiple vacations is investigated by Gao and Wang [6]. There are numerous research papers that have dealt with equilibrium balking strategies, one may refer to the monographs of Hassin [9].

The batch arrival occurrence is more frequent and sensible in the large-scale as the customers' service demand increases, such as transportation, tourism, etc. Guha et al. [8] studied equilibrium balking strategies in $GI^{[X]}/M/1$ queues with multiple and single working vacations. The present work is enhanced by considering a discrete time service process as the operation of real world communication devices are discrete in nature. To the best of our knowledge, there is no work reported regarding the infinite buffer discrete-time batch arrival renewal input queues with multiple working vacations. We investigate the arriving customers' balking behavior with respect to different level of information availability. We also compute the corresponding social net benefit of the fully observable and unobservable cases.

The rest of the paper is organized as follows. Section 2 gives the model description and assumptions of the reward-cost structure. The equilibrium balking and social optimal benefit strategies in case of fully observable are found in Section 3. Section 4 devotes to the equilibrium balking and social optimal benefit strategies in fully unobservable case. Section 5 presents the numerical results and compares the performances of fully observable and unobservable cases. Finally, Section 6 concludes the paper with possible future work.

2. Model Description and Assumptions

We consider a discrete-time renewal input batch arrival single server queueing system with multiple working vacations. Let us assume that customers arrive at the system in batches of random size X with probability mass function (pmf) $g_i = P(X = i), i \geq 1,$ probability generating function (pgf) $G(z) = \sum_{i=1}^{\infty} g_i z^i$ and mean batch size $\overline{g}$. The inter-arrival times $\{A_n, n \geq 1\}$ of batches are independent and identically distributed random variables with pmf $a_i = P(A_n = i)$, $i \geq 1$, corresponding pgf $A(z) = \sum_{i=1}^{\infty} a_i z^i$ and mean inter-arrival time $1/\lambda = A^{(1)}(1)$, where $A^{(n)}(k)$ is the n-th derivative of $A(z)$ with respect to z at $z = k$. When the system gets empty at the service completion epoch, the server takes multiple working vacations. Upon arrival of customers in a vacation period, the server serves the customers at a lower service rate. The vacation times V are independent and geometrically distributed with common pmf $P(V = i) = \overline{\phi}^{\,i-1}\phi, i \geq 1, 0 < \phi < 1,$ where for any real number $x \in [0,1]$, we denote $\overline{x} = 1 - x$. The service times in a regular busy period and in a working vacations period are independent and geometrically distributed with mean $1/\mu$ and $1/\eta$, respectively. An arriving batch with size larger than available buffer space is partly accepted and remaining part is rejected. Customers are served according to the first-come-first-served (FCFS) discipline.

We consider an early arrival system (EAS) setup. The time axis is assumed to be slotted into intervals of equal length with the length of a slot being unity. Let the time axis be marked by $0,1,2,\ldots,n,\ldots,$ and assume that potential arrival occurs in $(n,n+)$ and a potential departure takes place in $(n-,n)$. The working vacation can only start or end at n, that is, just after the departure. Consider the embedded points at the epochs immediately prior to the batch arrivals. Let successive batches of customers arrive at time epochs $\tau_0, \tau_1, \tau_2, \ldots$ and τ_n^- denote the time epochs just before the batch arrival instant τ_n. The inter-batch arrival times $T_{n+1} = \tau_{n+1} - \tau_n, n = 0,1,2, \ldots; T_0 = 0$ are mutually independent identically distributed random variables with common probability mass function $\{a_i, i \geq 1\}$. The state of the system at pre-arrival epoch of the k-th batch is defined as $\{N_s(\tau_k^-), J(\tau_k^-)\},$ where $N_s(\tau_k^-)$ denotes the number of customers in the system and $J(\tau_k^-) = 0$, indicates that the k-th batch arrival takes place during a working vacation period and $J(\tau_k^-) = 1$ indicates that the k-th batch arrival takes place during a regular service period. The process $\{N_s(\tau_k^-), J(\tau_k^-)\}$ is an embedded two-dimensional Markov chain with the state space $\Omega = \{(k,0), k \ge 0\} \bigcup \{(k,1), k \ge 1\}$. In limiting case,

$$P_{n,0}^- = \lim_{k\to\infty} P\{N_s(\tau_k^-) = n, J(\tau_k^-) = 0\}, n \ge 0,$$

$$P_{n,1}^- = \lim_{k\to\infty} P\{N_s(\tau_k^-) = n, J(\tau_k^-) = 1\}, n \ge 1,$$

where $P_{n,1}^- \, (P_{n,0}^-)$ denotes the probability that there are n customers in the system just before an arrival instant of a batch of customers when the server is in the regular service period (on working vacation).

We define the transition probability between two consecutive pre-arrival epochs as $P_{(m,i),(n,j)} = P\left\{N_s(\tau_k^-) = n, J(\tau_k^-) = j \mid N_s(\tau_{k-1}^-) = m, \xi(\tau_{k-1}^-) = i\right\}$. Let $P_{n,1} \, (P_{n,0})$ denotes the probability that there are n customers in the system when the server is in the regular service period (on working vacation) at an arbitrary epoch.

Our assumptions is that the decision to join or balk from the system is left upon the arriving batches. This decision is modeled by assuming that each customer receives a reward of R units upon completion of service and is charged a cost of C units per time unit during sojourn-time of the customer. We also assume that customers are risk neutral and wish to maximize their net benefit. At the arrival instant the customer computes the difference between expected waiting costs against their reward associated with receiving service.

Let b_j and c_j denote the probability that $j, j \ge 0$ customers have been served during an inter-batch arrival time when the service period is busy, and the working vacation continues, respectively. Let $d_j \, (j \ge 0)$ represent the probability that j customers have been served during an inter-batch arrival time given that working vacation terminates and service period is going on. Therefore, for all $j \ge 0$, we have

$$b_j = \sum_{i=j}^{\infty} a_i \binom{i}{j} \mu^j \bar{\mu}^{i-j}, \quad c_j = \sum_{i=max(1,j)}^{\infty} a_i \bar{\phi}^i \binom{i}{j} \eta^j \bar{\eta}^{i-j},$$

$$d_j = \sum_{i=max(1,j)}^{\infty} a_i \sum_{r=1}^{i} \bar{\phi}^{r-1} \phi \sum_{k=max(0,j+r-i)}^{min(j,r)} \binom{r}{k} \eta^k \bar{\eta}^{r-k} \binom{i-r}{j-k} \mu^{j-k} \bar{\mu}^{i-r-j+k}.$$

The probability generating function (p.g.f.) of b_j, c_j and d_j are given by

$$\tilde{B}(z) = \sum_{j=0}^{\infty} b_j z^j = A^*(\bar{\mu} + \mu z), \quad \tilde{C}(z) = \sum_{j=0}^{\infty} c_j z^j = A^*\left\{\bar{\phi}(\bar{\eta} + \eta z)\right\},$$

$$\tilde{D}(z) = \sum_{j=0}^{\infty} d_j z^j = \frac{\phi(\bar{\eta} + \eta z)\left[A^*(\bar{\mu} + \mu z) - A^*\left\{\bar{\phi}(\bar{\eta} + \eta z)\right\}\right]}{\bar{\mu} + \mu z - \bar{\phi}(\bar{\eta} + \eta z)}.$$

3. Analysis of Fully Observable Queues

In this section, we assume that the arriving batches are well informed about the queue length in addition to server state $(N_s(\tau_i^-), J(\tau_i^-))$ for fully observable case. Let us assume that the buffer capacity $n_e(0)$ (during working vacation) and $n_e(1)$ (during regular busy period). An equilibrium threshold strategy is specified by a pair $(n_e^*(0), n_e^*(1))$ and the balking strategy has the form 'Observe the embedded Markov chain $(N_s(\tau_i^-), J(\tau_i^-))$ at an arrival instant τ_i; enter if $N_s(\tau_i^-) < n_e^*(J(\tau_i^-))$ and balk otherwise'. In the following analysis, we restrict our attention to the existence of positive equilibrium, under which the server can be active. We have considered partial batch acceptance scheme. Due to the reduced rate of service during working vacation, we assume that the customers are not interested to join in a longer queue. Hence, we have considered $(n_e(0) < n_e(1))$. Our objective is to find out the values of $n_e^*(0)$ and $n_e^*(1)$ for which we can reach the equilibrium threshold which is defined in Section 3.4. Then $(N_s(\tau_i^-), J(\tau_i^-))$ is an embedded two-dimensional Markov chain with the state space $\Omega = \{(i,0); 0 \le i \le n_e(0)\} \cup \{(i,1); 1 \le i \le n_e(1)\}$.

The state probabilities at pre-arrival epochs, $P_{i,0}^-$'s ($0 \le i \le n_e(0)$) and $P_{i,1}^-$'s ($1 \le i \le n_e(1)$), may be obtained by solving the system of linear equations

$$(P_{i,0}^-, P_{i,1}^-) = \sum_{i=0}^{n_e(0)+n_e(1)+1} (P_{i,0}^-, P_{i,1}^-) Q_{ij}, \quad 0 \le j \le n_e(0)+n_e(1)+1,$$

where **Q**= [Q_{ij}] is the one-step transition probability matrix with four block matrices and is given below:

$$Q = \begin{bmatrix} VV_{(n_e(0)+1)\times(n_e(0)+1)} & VS_{(n_e(0)+1)\times(n_e(1))} \\ SV_{(n_e(1))\times(n_e(0)+1)} & SS_{(n_e(1))\times(n_e(1)} \end{bmatrix}_{(n_e(0)+n_e(1)+1)\times(n_e(0)+n_e(1)+1)}. \tag{1}$$

Their expressions for the partially and totally rejected models can be easily obtained using probabilistic arguments and are given below:

Blocks VV_{ij} and VS_{ij} refer to the transition from vacation state to vacation state and busy state, respectively and are given by the following expression:

$$VV_{ij} = \begin{cases} \sum_{k=j-i}^{n_e(0)-i} c_{i+k-j} g_k + c_{N-j} \sum_{k=n_e(0)-i+1}^{\infty} g_k, & j > i \ge 0, \\ \sum_{k=1}^{n_e(0)-i} c_{i+k-j} g_k + c_{n_e(0)-j} \sum_{k=n_e(0)-i+1}^{\infty} g_k, & 1 \le j \le i, \\ \psi_i & j = 0, 0 \le i \le n_e(0), \end{cases}$$

$$VS_{ij} = \begin{cases} \sum_{k=j-i}^{n_e(0)-i} d_{i+k-j} g_k + d_{n_e(0)-j} \sum_{k=n_e(0)-i+1}^{\infty} g_k, & j > i \ge 0, \\ \sum_{k=1}^{n_e(0)-i} d_{i+k-j} g_k + d_{n_e(0)-j} \sum_{k=n_e(0)-i+1}^{\infty} g_k, & 1 \le j \le i, \\ VS_{i-1,j}, & i = n_e(0), 1 \le j \le n_e(0), \end{cases}$$

where

$$\psi_i = 1 - \sum_{r=1}^{n_e(0)-i} g_r \sum_{k=0}^{i+r-1} (c_k + d_k) - \sum_{r=n_e(0)-i+1}^{\infty} g_r \sum_{k=0}^{n_e(0)-1} (c_k + d_k), 0 \le i \le n_e(0).$$

Similarly, the other blocks SV_{ij} and SS_{ij} refer to the transition from busy state to vacation state and busy state respectively, and are given by

$$SV_{ij} = \begin{cases} 1 - \sum_{r=1}^{n_e(1)-i} g_r \sum_{k=0}^{i+r-1} b_k - \sum_{r=n_e(1)-i+1}^{\infty} g_r \sum_{k=0}^{n_e(1)-1} b_k, & 1 \le i \le n_e(1), j = 0, \\ 0, & otherwise, \end{cases}$$

$$SS_{ij} = \begin{cases} \sum_{k=j-i}^{n_e(1)-i} b_{i+k-j} g_k + b_{n_e(1)-j} \sum_{k=n_e(1)-i+1}^{\infty} g_k, & j > i \ge 0, \\ \sum_{k=1}^{n_e(1)-i} b_{i+k-j} g_k + b_{n_e(1)-j} \sum_{k=n_e(1)-i+1}^{\infty} g_k, & 1 \le j \le i \\ SS_{i-1,j}, & i = n_e(1), 1 \le j \le n_e(1), \end{cases}$$

Let $Q = (Q_{ij})$ be the transition probability matrix (TPM), and $q^- = [P_{0,0}^-, P_{1,0}^-, \ldots, P_{n_e(0),0}^-, P_{1,1}^-, \ldots, P_{n_e(1),1}^-]$ be a row vector of pre-arrival epoch probabilities. Then q^- can be obtained by solving the system of equations $q^{-}Q = q^{-}$ using GTH algorithm [7].

A significant performance indicators of the batch arrival finite-buffer queueing system is the blocking probability of an arbitrary customer in an arriving batch (P_{BA}) is specified by

$$P_{BA} = \sum_{i=0}^{n_e(0)} P_{i,0}^{-} \sum_{j=n_e(0)-i}^{\infty} g_j^{-} + \sum_{i=1}^{n_e(1)} P_{i,1}^{-} \sum_{j=n_e(1)-i}^{\infty} g_j^{-},$$

where $g_i^{-} = \frac{1}{g}\sum_{j=i+1}^{\infty} g_j$, $i \geq 0$, refers the probability of i number of customers ahead of an arbitrary customer within the batch, see [5, p. 171].

3.1. Steady-state distribution at an arbitrary epoch

Let us derive the expression for steady-state system-length distribution at arbitrary epoch. The steady-state system-length distribution at an arbitrary epoch is obtained by using renewal theory [5]. Let $\hat{b}_k$ and $\hat{c}_k$ $(k \geq 0)$ be the probability that k customers have been served during a residual inter-batch arrival time when service period is in progress and working vacation continues, respectively. Similarly, let d_k be the probability that k customers have been served during a residual inter-batch arrival time given that working vacation has terminated and service period has began. Hence for all $k \geq 0$, we have

$$\hat{b}_j = \sum_{i=j}^{\infty} \binom{i}{j} \mu^j \overline{\mu}^{i-j} \lambda P\{T > i\}, \hat{c}_j = \sum_{i=max(1,j)}^{\infty} \overline{\phi}^i \binom{i}{j} \eta^j \overline{\eta}^{i-j} \lambda P\{T > i\},$$

$$\hat{d}_j = \sum_{i=max(1,j)}^{\infty} \sum_{r=1}^{i} \overline{\phi}^{r-1} \phi \sum_{k=max(0,j+r-1)}^{min(j,r)} \binom{r}{k} \eta^k \overline{\eta}^{r-k} \binom{i-r}{j-k} \mu^{j-k} \overline{\mu}^{i-r-j+k} \lambda P\{T > i\}.$$

System-length at an arbitrary epoch $P_{i,0}, P_{i,1}$ can be obtained from Q by applying $\hat{b}_j$, $\hat{c}_j$ and $\hat{d}_j$ in place of b_j, c_j and d_j in (1). Then $q = [P_{0,0}, P_{1,0}, \ldots, P_{n_e(0),0}, P_{1,1}, \ldots, P_{n_e(1),1}]$ can be obtained by solving the system of equations $qQ = q$ using GTH algorithm [7].

3.2. Outside observer's distribution

The outside observer's observation epoch falls in a time interval after a potential arrival and before a potential departure. Let $P_{i,0}^{o}$ $(0 \leq i \leq n_e(0))$ and $P_{i,1}^{o}$ $(0 \leq i \leq n_e(1))$ denote the probabilities that i customers are in the system when the outside observer sees the server on working vacation and in the busy period, respectively. They are given by

$$P_{0,0} = P_{0,0}^{o} + \eta P_{1,0}^{o} + \mu P_{1,1}^{o}, \tag{2}$$

$$P_{i,0} = \overline{\phi}\overline{\eta} P_{i,0}^{o} + \overline{\phi}\eta P_{i+1,0}^{o}, 1 \leq i \leq n_e(0) - 1, \tag{3}$$

$$P_{n_e(0),0} = \overline{\phi}\overline{\eta} P_{n_e(0),0}^{o}, \tag{4}$$

$$P_{i,1} = \overline{\mu}\pi_{i,1}^{o} + \mu P_{i+1,1}^{o} + \phi\overline{\eta} P_{i,0}^{o} + \phi\eta P_{i+1,0}^{o}, 1 \leq i \leq n_e(0) - 1, \tag{5}$$

$$P_{n_e(0),1} = \overline{\mu} P_{n_e(0),1}^{o} + \mu P_{n_e(0)+1,1}^{o} + \phi\overline{\eta} P_{n_e(0),0}^{o}, \tag{6}$$

$$P_{i,1} = \overline{\mu} P_{i,1}^{o} + \mu P_{i+1,1}^{o}, n_e(0) + 1 \leq i \leq n_e(1) - 1, \tag{7}$$

$$P_{n_e(1),1} = \overline{\mu} P_{n_e(1),1}^{o}. \tag{8}$$

Subsequently $P_{i,0}^{o}$ and $P_{i,1}^{o}$ can be obtained from the above (2)-(8).

3.3. Sojourn time analysis

We compute the sojourn time in system for an arbitrary customer in an arriving batch. Let $E[W_A]$

be the mean sojourn time in the system for an arbitrary customer and $W_A^*(s)$ denote the probability generating function (pgf) of the sojourn time for the same. We can compute $W_A^*(z)$ for partial batch acceptance model as:

$$W_A^*(z) = \frac{1}{(1-P_{BA})}\Big[\sum_{j=0}^{n_e(0)-1} P_{j,0}^- \sum_{r=0}^{n_e(0)-j-1} g_r^- \frac{1}{\bar{\phi}}\left(\frac{\eta\bar{\phi}z}{1-\bar{\phi}\bar{\eta}z}\right)^{r+j+1} + \sum_{j=0}^{n_e(0)-1} P_{j,0}^- \sum_{r=0}^{n_e(0)-j-1} g_r^-$$
$$\times\left\{\left(\frac{\phi\bar{\eta}z}{1-\bar{\phi}\bar{\eta}z}\right)\left(\frac{\mu z}{1-\bar{\mu}z}\right)^{r+j+1} + \sum_{k=1}^{r+j}\frac{1}{\bar{\phi}}\left(\frac{\eta\bar{\phi}z}{1-\bar{\phi}\bar{\eta}z}\right)^{k}\left(\frac{\phi}{1-\bar{\phi}\bar{\eta}z}\right)\left(\frac{\mu z}{1-\bar{\mu}z}\right)^{r+j-k+1}\right.$$
$$\left. + \sum_{j=1}^{n_e(1)-1} P_{j,1}^- \sum_{r=0}^{n_e(1)-j-1} g_r^- \left(\frac{\mu z}{1-\bar{\mu}z}\right)^{r+j+1}\right\}\Big]. \quad (9)$$

Now the mean sojourn times in the system for an arbitrary customer ($E[W_A]$) may be computed as

$$E[W_A] = \frac{1}{1-P_{BA}}\Big[\sum_{j=0}^{n_e(0)-1} P_{j,0}^- \sum_{r=0}^{n_e(0)-j-1} g_r^- \left\{\frac{r+j+1}{\eta\bar{\phi}^2}\left(\frac{\eta\bar{\phi}}{1-\bar{\phi}\bar{\eta}}\right)^{r+j+2}\right.$$
$$\left. + \frac{\phi\bar{\eta}[\mu+(j+r+1)(1-\bar{\phi}\bar{\eta})]}{\mu(1-\bar{\phi}\bar{\eta})^2} + \sum_{k=1}^{r+j}\frac{\phi\bar{\phi}^{k-1}\eta^k[(k+\bar{\phi}\bar{\eta})\mu+(j+r+1-k)(1-\bar{\phi}\bar{\eta})]}{\mu(1-\bar{\phi}\bar{\eta})^{k+2}}\right\}$$
$$+ \sum_{j=1}^{n_e(1)-1} P_{j,1}^- \sum_{r=0}^{n_e(1)-j-1} g_r^- \left(\frac{r+j+1}{\mu}\right)\Big]. \quad (10)$$

We have considered $E[W_A]$ for our numerical purposes in the rest of the paper.

3.4. Balking strategies

3.4.1. Relation between reward and cost

Each customer has to pay a fixed cost C after completing service. The reward-cost is chosen in such a manner that the customer is attracted to participate in the system even when the system is empty at the batch arrival instant whose probability is $P_{0,0}^-$. Considering $i=0$ in (10), the mean sojourn time of an arbitrary customer in a batch who arrives at an empty system (denoted by Δ_{fobs}) is given by

$$\Delta_{fobs} = \sum_{r=0}^{n_e(0)-1} g_r^- \left\{\frac{r+1}{\eta\bar{\phi}^2}\left(\frac{\eta\bar{\phi}}{1-\bar{\phi}\bar{\eta}}\right)^{r+2} + \frac{\phi\bar{\eta}[(r+1)(1-\bar{\phi}\bar{\eta})+\mu]}{\mu(1-\bar{\phi}\bar{\eta})^2}\right.$$
$$\left. + \sum_{k=1}^{r}\frac{\phi\bar{\phi}^{k-1}\eta^k[(k+\bar{\phi}\bar{\eta})\mu+(r+1-k)(1-\bar{\phi}\bar{\eta})]}{\mu(1-\bar{\phi}\bar{\eta})^{k+2}}\right\}. \quad (11)$$

The system should be designed in such a way that reward (R) shall not be very high and cost (C) may not be very less for attracting too many customers to participate in the system. The linear relationship between R and C should follow the constraint given below

$$R > C\cdot\Delta_{fobs}. \quad (12)$$

3.4.2. Equilibrium balking strategy

Let $\Delta = R - C\cdot E[W_A]$ be the net benefit of a customer who has been served. Let us define the equilibrium threshold ($n_e^*(0)$ and $n_e^*(1)$) as the number of customers in the system when an arriving customer finds the net benefit (Δ) becomes zero or closest to zero. Under low traffic intensity the system-length never grows beyond a particular value. In that case, the particular value of system-length for which the system occupancy never grows beyond a threshold is known as equilibrium threshold. Under high traffic intensity the numerical results show that there exists an equilibrium threshold when $\Delta = 0$.

3.4.3. Socially optimal balking strategy

Evaluation of steady-state distribution at arbitrary epoch is needed for computation of social benefit per unit time. The average number of customers in the system at an outside observer's observation epoch (L_s^o) is given by $L_s^o = \sum_{i=1}^{n_e(0)} iP_{i,0}^o + \sum_{i=1}^{n_e(1)} iP_{i,1}^o$. We may evaluate the average number of customers in the system at an arbitrary epoch (L_s) by replacing $P_{i,0}^o\ (P_{i,1}^o)$ by $P_{i,0}\ (P_{i,1})$. The average waiting-time in the system may be computed using Little's rule as $E[W_A] = L_s^o / \lambda_e$, where $\lambda_e = \lambda \bar{g}(1-P_{BA})$ is the effective arrival rate. Let $\Delta_s(n_e(0), n_e(1)$ be the social benefit per time unit, which can be expressed as

$$\Delta_s(n_e(0), n_e(1)) = R\lambda\bar{g}(1-P_{BA}) - C\left(\sum_{i=1}^{n_e(0)} iP_{i,0}^o + \sum_{i=1}^{n_e(1)} iP_{i,1}^o\right).$$

4. Analysis of Fully Unobservable Queues with Balking

An arriving batch remains unaware about the system-length while deciding whether to join or balk for unobservable queues. As a result, unobservable queue behaves as an infinite buffer $GI^{[X]}/Geo/1$ queueing system. Unobservable queues are classified under two categories: (1) Almost unobservable case: The status of the server $\xi(t)$ is known to the arriving batches; (2) Fully unobservable case: Information is not made available to the arriving batches to distinguish whether server is on vacation or on normal busy period. Let f be the joining probability of the arriving batches to the system. The following difference equations are obtained by observing the state of the system at two consecutive pre-arrival epochs:

$$P_{0,0}^- = fP_{0,0}^- \sum_{r=1}^{\infty} g_r \sum_{i=r}^{\infty}(c_i + d_i) + (1-f)P_{0,0}^- + f\sum_{i=1}^{\infty} P_{i,1}^- \sum_{r=1}^{\infty} g_r \sum_{k=i+r}^{\infty} b_k + (1-f)\sum_{i=1}^{\infty} P_{i,1}^- \sum_{k=i}^{\infty} b_k$$
$$+ f\sum_{i=1}^{\infty} P_{i,0}^- \sum_{r=1}^{\infty} g_r \sum_{k=i+r}^{\infty}(c_k + d_k) + (1-f)\sum_{i=1}^{\infty} P_{i,0}^- \sum_{k=i}^{\infty}(c_k + d_k), \tag{13}$$

$$P_{1,0}^- = f\sum_{k=0}^{\infty} P_{k,0}^- \sum_{r=1}^{\infty} g_r c_{k+r-1} + (1-f)\sum_{k=1}^{\infty} P_{k,0}^- c_{k-1}, \tag{14}$$

$$P_{i,0}^- = f\sum_{k=0}^{i-2} P_{k,0}^- \sum_{r=i-k}^{\infty} g_r c_{k+r-i} + f\sum_{k=i-1}^{\infty} P_{k,0}^- \sum_{r=1}^{\infty} g_r c_{k+r-i} + (1-f)\sum_{k=i}^{\infty} P_{k,0}^- c_{k-i}, i \geq 2, \tag{15}$$

$$P_{1,1}^- = f\sum_{k=0}^{\infty} P_{k,0}^- \sum_{r=1}^{\infty} g_r d_{k+r-1} + (1-f)\sum_{k=1}^{\infty} P_{k,0}^- d_{k-1} + f\sum_{k=1}^{\infty} P_{k,1}^- \sum_{r=1}^{\infty} g_r b_{k+r-1}$$
$$+(1-f)\sum_{k=1}^{\infty} P_{k,1}^- b_{k-1}, \tag{16}$$

$$P_{i,1}^- = f\left(\sum_{k=0}^{i-2} P_{k,0}^- \sum_{r=i-k}^{\infty} g_r d_{k+r-i} + \sum_{k=i-1}^{\infty} P_{k,0}^- \sum_{r=1}^{\infty} g_r d_{k+r-i} + \sum_{k=1}^{i-2} P_{k,1}^- \sum_{r=i-k}^{\infty} g_r b_{k+r-i}\right.$$
$$\left. + \sum_{k=i-1}^{\infty} P_{k,1}^- \sum_{r=1}^{\infty} g_r b_{k+r-i}\right) + (1-f)\left(\sum_{k=i}^{\infty} P_{k,1}^- b_{k-i} + \sum_{k=i}^{\infty} P_{k,0}^- d_{k-i}\right), i \geq 2. \tag{17}$$

Summing (13)-(15), after multiplying the appropriate powers of z^i and using the definition of pgf $P_0^{-*}(z) = \sum_{i=0}^{\infty} P_{i,0}^- z^i$, we get (18). In similar fashion, multiplying (16) and (17) by the appropriate powers of z^{i-1}, and applying the definition of pgf, $P_1^{-*}(z)$ as $P_1^{-*}(z) = \sum_{i=1}^{\infty} P_{i,1}^- z^{i-1}$, we obtain (19).

$$P_0^{-*}(z) = \frac{P_{0,0}^- - f\sum_{i=0}^{\infty} P_{i,0}^- z^i \sum_{j=i+1}^{\infty} \frac{c_j}{z^j} \sum_{r=1}^{j-i} g_r z^r - (1-f)\sum_{i=0}^{\infty} P_{i,0}^- z^i \sum_{j=i}^{\infty} \frac{c_j}{z^j}}{1-(1-f+fG(z))\overline{C}(\frac{1}{z})}, \tag{18}$$

$$P_1^{-*}(z)=\frac{(1-f+fG(z))\frac{1}{z}\overline{D}(\frac{1}{z})P_0^{-*}(z)-f\sum_{i=0}^{\infty}P_{i,0}^{-}z^{i-1}\sum_{j=i+1}^{\infty}\frac{d_j}{z^j}\sum_{r=1}^{j-i}g_r z^r}{1-(1-f+fG(z))\overline{B}(\frac{1}{z})}$$
$$-\frac{(1-f)\sum_{i=0}^{\infty}P_{i,0}^{-}z^{i-1}\sum_{j=i}^{\infty}\frac{d_j}{z^j}+f\sum_{i=1}^{\infty}P_{i,1}^{-}z^{i-1}\sum_{j=i}^{\infty}\frac{b_j}{z^j}\sum_{r=1}^{j-i}g_r z^r}{1-(1-f+fG(z))\overline{B}(\frac{1}{z})}$$
$$-\frac{(1-f)\sum_{i=1}^{\infty}P_{i,1}^{-}z^{i-1}\sum_{j=i}^{\infty}\frac{b_j}{z^j}}{1-(1-f+fG(z))\overline{B}(\frac{1}{z})}. \tag{19}$$

The above expressions are analytic and convergent in $|z|\leq 1$. We require to compute zeros of the denominator (18) and (19). Let us assume that the maximum size of an arriving batch is $\hat{r}$. The equation $1-(1-f+fG(z))\overline{C}(\frac{1}{z})=0$ and $1-(1-f+fG(z))\overline{B}(\frac{1}{z})=0$ has exactly $\hat{r}$ roots in the region $|z|>1$, see ([5], pp. 119). We consider the $\hat{r}$ roots of the equations $1-(1-f+fG(z))\overline{B}(\frac{1}{z})=0$ and $1-(1-f+fG(z))\overline{C}(\frac{1}{z})=0$, respectively, in the region $|z|>1$. As the denominator of (18) has $\hat{r}$ roots outside the unit circle, the function $1-(1-f+fG(\frac{1}{z}))\overline{C}(z)=0$ has $\hat{r}$ zeros ω_i inside the unit circle $|z|<1$. As $P_0^{-*}(z)$ is an analytic function of z for $|z|\leq 1$, employing the partial-fraction method, we get

$$P_0^{-*}(z)=\sum_{i=1}^{\hat{r}}\frac{H_i}{1-\omega_i z}, \tag{20}$$

where H_i are the $\hat{r}$ constants to be determined. In order to simplify the expression of $P_1^{-*}(z)$, let us consider

$$T(z)=-\frac{f\sum_{i=0}^{\infty}P_{i,0}^{-}z^{i-1}\sum_{j=i+1}^{\infty}\frac{d_j}{z^j}\sum_{r=1}^{j-i}g_r z^r+(1-f)\sum_{i=0}^{\infty}P_{i,0}^{-}z^{i-1}\sum_{j=i}^{\infty}\frac{d_j}{z^j}}{1-(1-f+fG(z))\overline{B}(\frac{1}{z})}$$
$$-\frac{f\sum_{i=1}^{\infty}P_{i,1}^{-}z^{i-1}\sum_{j=i}^{\infty}\frac{b_j}{z^j}\sum_{r=1}^{j-i}g_r z^r+(1-f)\sum_{i=1}^{\infty}P_{i,1}^{-}z^{i-1}\sum_{j=i}^{\infty}\frac{b_j}{z^j}}{1-(1-f+fG(z))\overline{B}(\frac{1}{z})}.$$

Then $P_1^{-*}(z)$ can be further simplified as

$$P_1^{-*}(z)=\frac{(1-f+fG(z))\frac{1}{z}\overline{D}(\frac{1}{z})P_0^{-*}(z)}{1-(1-f+fG(z))\overline{B}(\frac{1}{z})}+T(z). \tag{21}$$

Now let us analyze the analytic expression (21). Since the equation $1-(1-f+fG(z))\overline{B}(\frac{1}{z})=0$ has $\hat{r}$ roots outside the unit circle, the function $1-(1-f+fG(\frac{1}{z}))\overline{B}(z)=0$ has $\hat{r}$ zeros ξ_i inside the unit circle. It is obvious that $P_1^{-*}(z)$ is depending on $P_0^{-*}(z)$. As $P_1^{-*}(z)$ is an analytic function of z for $|z|\leq 1$, applying the partial-fraction method, we obtain

$$P_1^{-*}(z)=\sum_{i=1}^{\hat{r}}\frac{K_i}{1-\omega_i z}+\sum_{i=1}^{\hat{r}}\frac{L_i}{1-\xi_i z}. \tag{22}$$

Now, equating the coefficient of z^i from both sides of (20) and (22), the pre-arrival epoch probabilities can be obtained as

$$P_{k,0}^{-} = \sum_{i=1}^{\hat{r}} H_i \omega_i^k, k \geq 0, \tag{23}$$

$$P_{k,1}^{-} = \sum_{i=1}^{\hat{r}} K_i \omega_i^{k-1} + \sum_{i=1}^{\hat{r}} L_i \xi_i^{k-1}, k \geq 1. \tag{24}$$

At this point, the unknown constants are H_i, K_i, and $L_i (i = 1, 2, \hat{r})$. Considering Eq. (13)-(15), for $i = 2, 3, \ldots, \hat{r} - 1$, (16) and (17) for $i = 2, 3, 2\hat{r} - 1$, we have $3\hat{r}$ simultaneous equations with $3\hat{r}$ unknowns. One may note here that we ignore the values for $i \geq \hat{r}$ of (15) which are redundant equations and we must use normalizing condition by putting $z = 1$ in (20) and (21) which is given below as

$$\sum_{i=1}^{\hat{r}} \frac{H_i}{1-\omega_i} + \sum_{i=1}^{\hat{r}} \frac{K_i}{1-\omega_i} + \sum_{i=1}^{\hat{r}} \frac{L_i}{1-\xi_i} = 1.$$

Therefore, solving these $3\hat{r}$ equations, we can evaluate $3\hat{r}$ unknown constants.

4.1. Steady-state distribution at an arbitrary epoch

System-length at an arbitrary epoch $P_{i,0}, P_{i,1}$ can be obtained from pre-arrival epoch probabilities $P_{i,0}^{-}, P_{i,1}^{-}$ $(\forall i \geq 1)$ by employing $\hat{b}_j$, $\hat{c}_j$ and $\hat{d}_j$ in place of b_j, c_j and d_j in (14)- (17), which is described below

$$P_{1,0} = f \sum_{k=0}^{\infty} P_{k,0}^{-} \sum_{r=1}^{\infty} g_r \hat{c}_{k+r-1} + (1-f) \sum_{k=1}^{\infty} P_{k,0}^{-} \hat{c}_{k-1} \tag{25}$$

$$P_{i,0} = f\left(\sum_{k=0}^{i-2} P_{k,0}^{-} \sum_{r=i-k}^{\infty} g_r \hat{c}_{k+r-i} + \sum_{k=i-1}^{\infty} P_{k,0}^{-} \sum_{r=1}^{\infty} g_r \hat{c}_{k+r-i}\right) + (1-f) \sum_{k=i}^{\infty} P_{k,0}^{-} \hat{c}_{k-i}, i \geq 2, \tag{26}$$

$$P_{1,1} = f \sum_{k=0}^{\infty} P_{k,0}^{-} \sum_{r=1}^{\infty} g_r d_{k+r-1} + (1-f) \sum_{k=1}^{\infty} P_{k,0}^{-} d_{k-1} + \sum_{k=1}^{\infty} P_{k,1}^{-} \left\{ f \sum_{r=1}^{\infty} g_r \hat{b}_{k+r-1} + (1-f) \hat{b}_{k-1} \right\}, \tag{27}$$

$$P_{i,1} = f\left(\sum_{k=0}^{i-2} P_{k,0}^{-} \sum_{r=i-k}^{\infty} g_r d_{k+r-i} + \sum_{k=i-1}^{\infty} P_{k,0}^{-} \sum_{r=1}^{\infty} g_r d_{k+r-i}\right) + f \sum_{k=1}^{i-2} P_{k,1}^{-} \sum_{r=i-k}^{\infty} g_r \hat{b}_{k+r-i}$$
$$+ f \sum_{k=i-1}^{\infty} P_{k,1}^{-} \sum_{r=1}^{\infty} g_r \hat{b}_{k+r-i} + (1-f) \sum_{k=i}^{\infty} P_{k,1}^{-} \hat{b}_{k-i} + (1-f) \sum_{k=i}^{\infty} P_{k,0}^{-} d_{k-i}, i \geq 2. \tag{28}$$

After obtaining $P_{i,0}$ and $P_{i,1}$ $(\forall i \geq 1)$, one can obtain $P_{0,0} = 1 - \sum_{i=1}^{\infty} (P_{i,0} + P_{i,1})$.

4.2. Outside observer's distribution

Similarly the outside observer's distribution in case of fully unobservable case may be obtained using (2), (3), and (5). Let L_s^o denotes mean system-length that can be obtained from steady-state distribution at arbitrary epoch as $L_s^o = \sum_{i=1}^{\infty} i (P_{i,0}^o + P_{i,1}^o)$. Considering $f = 1, g_1 = 1$ and $g_i = 0, \forall i \geq 2$, our model reduces to $GI/Geo/1/MWV$ queueing without balking [11].

4.3. Sojourn-time analysis

Waiting time in the system can be computed from the pre-arrival epoch probabilities $P_{i,0}^{-}$ and $P_{i,1}^{-}$ by following the similar derivation of (14)-(17). The unobservable balking behaves similarly as an infinite buffer queueing system, we focus on the sojourn time for an arbitrary customer in an arriving batch. Hence, $W_A^*(z)$ can be given by,

$$W_A^*(z) = 1 - f + f \sum_{j=0}^{\infty} P_{j,0}^{-} \sum_{r=0}^{\infty} g_r^{-} \left\{ \frac{1}{\bar{\phi}} \left(\frac{\eta \bar{\phi} z}{1 - \bar{\phi} \bar{\eta} z} \right)^{r+j+1} + \left(\frac{\phi \bar{\eta} z}{1 - \bar{\phi} \bar{\eta} z} \right) \left(\frac{\mu z}{1 - \bar{\mu} z} \right)^{r+j+1} \right.$$

$$+\sum_{k=1}^{r+j}\frac{1}{\overline{\phi}}\left(\frac{\eta\overline{\phi}z}{1-\overline{\phi}\overline{\eta}z}\right)^{k}\left(\frac{\phi}{1-\overline{\phi}\overline{\eta}z}\right)\left(\frac{\mu z}{1-\overline{\mu}z}\right)^{r+j-k+1}\Bigg\}+\sum_{j=1}^{\infty}P_{j,1}^{-}\sum_{r=0}^{\infty}g_r^{-}\left(\frac{\mu z}{1-\overline{\mu}z}\right)^{r+j+1}. \tag{29}$$

The mean of the sojourn time of an arbitrary customer ($E[W_A]$) of an arriving batch is given by

$$E[W_A]=f\sum_{j=0}^{\infty}P_{j,0}^{-}\sum_{r=0}^{\infty}g_r^{-}\left\{\frac{r+j+1}{\eta\overline{\phi}^2}\left(\frac{\eta\overline{\phi}}{1-\overline{\phi}\overline{\eta}}\right)^{r+j+2}+\frac{\phi\overline{\eta}[(j+r+1)(1-\overline{\phi}\overline{\eta})+\mu]}{\mu(1-\overline{\phi}\overline{\eta})^2}\right.$$

$$\left.+\sum_{k=1}^{r+j}\frac{\phi\overline{\phi}^{k-1}\eta^k[(k+\overline{\phi}\overline{\eta})\mu+(j+r+1-k)(1-\overline{\phi}\overline{\eta})]}{\mu(1-\overline{\phi}\overline{\eta})^{k+2}}\right\}+f\sum_{j=1}^{\infty}P_{j,1}^{-}\sum_{r=0}^{\infty}g_r^{-}\frac{r+j+1}{\mu}. \tag{30}$$

4.4. Balking strategies

4.4.1. Equilibrium balking strategy

Let Δ_{funobs} be the mean waiting for an arbitrary customer in a batch that finds the system to be empty at arrival. Using similar derivation of Section 3.4 and considering $i=0$ in (30), Δ_{funobs} is given by

$$\Delta_{funobs}=f\sum_{r=0}^{\infty}g_r^{-}\left\{\frac{r+1}{\eta\overline{\phi}^2}\left(\frac{\eta\overline{\phi}}{1-\overline{\phi}\overline{\eta}}\right)^{r+2}+\frac{\phi\overline{\eta}[(r+1)(1-\overline{\phi}\overline{\eta})+\mu]}{\mu(1-\overline{\phi}\overline{\eta})^2}\right.$$

$$\left.+\sum_{k=1}^{r}\left(\frac{\phi\overline{\phi}^{k-1}\eta^k[(r+1)(1-\overline{\phi}\overline{\eta})+(k+1)\overline{\phi}\overline{\eta}]}{(1-\overline{\phi}\overline{\eta})^{k+2}}+\frac{(r+1-k)\phi\overline{\phi}^{k-1}\eta^k\overline{\mu}}{\mu(1-\overline{\phi}\overline{\eta})^{k+1}}\right)\right\}. \tag{31}$$

A linear relation between cost and reward can be obtained from (12) by replacing Δ_{fobs} by Δ_{funobs}. The net benefit $\Delta_e(f)=R-C\cdot E[W_A]$ is depending upon mean sojourn time in the system. The joining probability (f) which produces Δ closest to zero is the desired equilibrium joining probability for the fully unobservable queueing system. The pair of values of (f,f) which produces $\Delta_e(f,f)$ closest to zero are the desired equilibrium joining probability (denoted as $f_e^*(0)$ and $f_e^*(1)$) when the system is in working vacation or normal busy period, respectively. Let us denote the net benefit for equilibrium balking case as $\Delta_e(d)=R-C\cdot E[W_A]$. Solving $\Delta_e(f)$ where $f\in[0,\frac{\mu}{\lambda\overline{g}})$ and taking $\frac{\mu}{\lambda\overline{g}}<1$, we get the positive and feasible root f_e^*. For fully unobservable case, the equilibrium joining probability is denoted as f_e^*. We numerically observe that $E[W_A]$ is increasing with $\lambda\overline{g}(\lambda\overline{g}<\mu)$. It can be seen that the customer's mixed strategy f_e is unique in the unobservable queues and $f_e=min\{f_e^*,1\}$.

4.4.2. Socially optimal balking strategy

For the social planner, we impose a mixed strategy f so that social benefit per time unit may be maximized. Using Little's law, $\Delta_s(f)$ is given by

$$\Delta_s(f)=\lambda\overline{g}f(R-C\cdot E[W_A]). \tag{32}$$

The first order and the second order derivatives of $\Delta_s(f)$ with respect to f are given as

$$\Delta_s'(f)=\lambda\overline{g}\left(R-C\cdot E[W_A]\right)-\lambda\overline{g}fC\cdot E'[W_A],\ [\because E'[W_A]=\frac{d}{df}E[W_A]]$$

$$\Delta_s''(f)=-\lambda\overline{g}C\cdot E'[W_A]-\lambda\overline{g}C\cdot E'[W_A]=-2\lambda\overline{g}C\cdot E'[W_A].$$

In order to get first-order optimality conditions, we have to take $\Delta_s'(f)=0$, that is,

$$\frac{R}{C}-\left(E[W_A]+f\cdot E'[W_A]\right)=0. \tag{33}$$

By recalling from (30) that $E[W_A]=f\cdot\Psi$ where

$$\Psi = \sum_{j=0}^{\infty} P_{j,0}^{-} \sum_{r=0}^{\infty} g_r^{-} \left\{ \frac{r+j+1}{\eta\bar{\phi}^2} \left(\frac{\eta\bar{\phi}}{1-\bar{\phi}\bar{\eta}} \right)^{r+j+2} + \frac{\phi\bar{\eta}[(j+r+1)(1-\bar{\phi}\bar{\eta})+\mu]}{\mu(1-\bar{\phi}\bar{\eta})^2} \right.$$
$$\left. + \sum_{k=1}^{r+j} \frac{\phi\bar{\phi}^{k-1}\eta^k[(k+\bar{\phi}\bar{\eta})\mu+(j+r+1-k)(1-\bar{\phi}\bar{\eta})]}{\mu(1-\bar{\phi}\bar{\eta})^{k+2}} \right\} + \sum_{j=1}^{\infty} P_{j,1}^{-} \sum_{r=0}^{\infty} g_r^{-} \frac{r+j+1}{\mu}.$$

Let f^* is the socially optimal joining probability in the unobservable queue. It is easily seen that $f = f^*$ is the solution of $\Delta'_s(f) = 0$, by solving (33), f^* can be obtained by using $f^* = \frac{R}{2C \cdot \Psi}$.

As $E'[W_A] \geq 0$, we have that $\Delta''_s(f) < 0$ for any probability $[0, \frac{\mu}{\lambda \bar{g}})$. Therefore, the function $\Delta_s(f)$ in (32) is concave function of f in this interval, and it attains a unique maximum at the point $f = f^*$. If $f^* < 1$, then the maximum of $\Delta_s(f)$ in the interval $[0, \frac{\mu}{\lambda \bar{g}})$ is attained at $f = f^*$; otherwise, it is attained at $f = 1$.

5. Numerical Results

In this section, we present the effect of the system parameters on the equilibrium and the social benefit for both fully observable and unobservable cases. The parameters are assumed as $\lambda = 0.2,\ \theta = 0.3,\ \eta = 0.4, R = 20,\ C = 8,\ 0.3 \leq \mu \leq 0.8$ and batch size distribution as $g_1 = 0.6,\ g_2 = 0.6,\ g_3 = 0.1, g_4 = 0.1$. Figure 1 presents the impact of the mean vacation time (ϕ) on the mean waiting time in the system ($E[W_A]$) when the inter-batch arrival time distribution is deterministic. It is observed that for all values of traffic load (ρ), the mean waiting-time in the system decreases as the mean vacation time increases. We further observe that for fixed mean vacation time, the mean waiting-time in the system increases as traffic load increases. Subsequently in the following discussion we consider the inter-batch arrival time is following an arbitrary distribution. Figure 2 illustrates the dependence of the mean waiting time in the system on the traffic load (ρ) for various mean vacation time. It is observed that the mean waiting time in the system increases as traffic load increases. Moreover, $E[W_A]$ decreases as the mean vacation time increases.

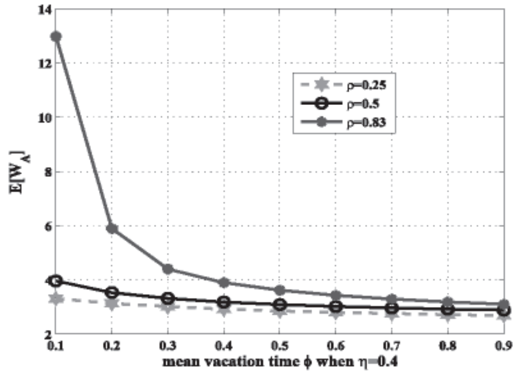

Figure 1. ϕ vs $E[W_A]$ for deterministic inter-arrival time in fully observable queue

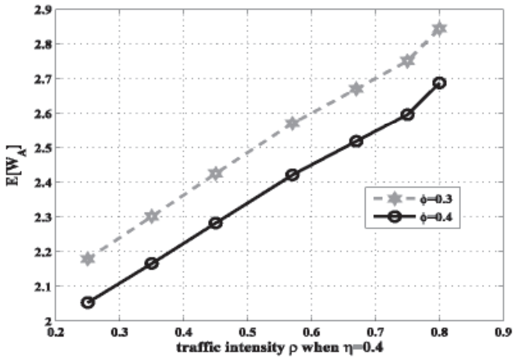

Figure 2. ρ vs $E[W_A]$ for inter-arrival following an arbitrary distribution in fully unobservable queue

Figures 3 and 4 depict the behavior of the net benefit and social benefit with respect to joining probability for a fully unobservable case, when the inter-batch arrival is following a geometric distribution. The net benefit increases slowly upon the increment of joining probability up to certain level, thereafter the net benefit decreases sharply with further increase of joining probability. The equilibrium joining probability is shown in Figure 3 when the net benefit becomes zero or closest to zero. Similarly, the social joining probability is also shown in Figure 4. The net benefit and social benefit decreases with the increase in the values of f. It is seen that for $\mu = 0.3$ and $\mu = 0.4$, there exists a point f in the interval $(0,1)$ such that $\Delta = 0$ and $\Delta_s(f) = 0$. The value of Δ and $\Delta_s(f)$ are positive or zero or negative according to the value of f is less than or equal to or greater than 0.75. If $\mu \geq 0.5$, both benefits are always positive for any $f \in (0,1)$. Therefore, the optimal response for individual customer is to join the queue with probability one, that is, $f_e = 1$ when $\mu \geq 0.5$.

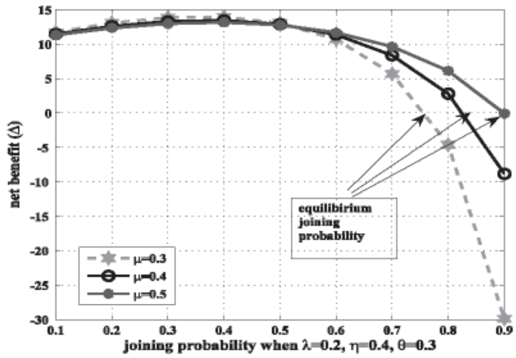

Figure 3. Joining probability vs net benefit for fully unobservable queue

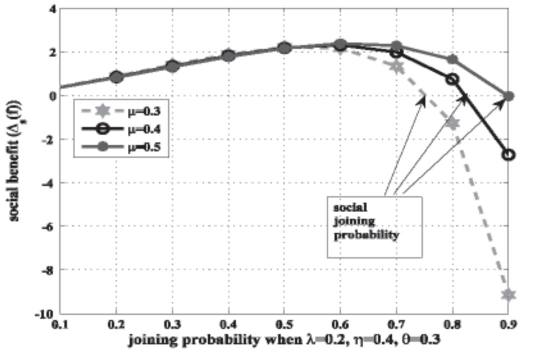

Figure 4. Joining probability vs social benefit for fully unobservable queue

6. Conclusions

An equilibrium balking strategies in discrete-time $GI^X/Geo/1$ queueing systems with multiple working vacations have been studied from an economic point of view. We considered the stationary behavior of the systems for fully observable and unobservable cases, and derived the equilibrium balking and social optimal strategies. Customers can take their decisions according to the exact situation, which is more practical than the classical assumptions in queueing theory. We have discussed the fully observable and unobservable cases with reference to the level of information given to arriving customers and found the equilibrium strategies. The closed form expressions to obtain the probability generating functions of the system-length at the pre-arrival epoch in the fully unobservable case have been provided. Moreover, the equilibrium balking and social benefit behavior with reference to various parameters on the impact of the information level have been carried out. Analysis of equilibrium joining with state dependent balking and reneging for $GI^X/Geo/1$ queueing systems can be carried out as future works. Furthermore, there is a necessity to look into operational controls, such as scheduling.

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